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A "SNOWFLAKE" DIVERTOR AND ITS PROPERTIES

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Handling the power and particle exhaust in fusion reactors based on tokamaks is a challenging problem [1,2]. To bring the energy flux to the divertor plates to an acceptable level (< 10 MW/m^2), it is desirable to significantly increase poloidal flux expansion in the divertor area. Some recent ideas include that of a so-called X divertor [3] and a "snowflake" divertor [4]. We use an acronym SF to designate the latter.

In this paper we concentrate on the SF divertor. The general idea behind this configuration is that, by a proper selection of divertor (poloidal field) coils, one can make the null point of the second, not of the first order as in the standard divertor. The separatrix in the vicinity of the X point then acquires a characteristic hexapole structure (Fig. 1), reminiscent of a snowflake, whence the name. The fact that the field has a second-order null, leads to a significant increase of the flux expansion.

It was noted in Ref. [4] that the SF configuration is topologically unstable: if the current in the divertor coils is somewhat higher than the one that provides the SF configuration, it becomes a single-null X-point configuration. Conversely, if the coil current becomes somewhat lower, there appear two separate X-points. To solve this problem, one can operate the divertor at the current by roughly 5 % higher than the value needed to create the second-order null. Then, configuration becomes robust enough and the shape of the separatrix does not change significantly if the coil current varies by 2-3 %. At the same time, the flux expansion still remained by a factor of ~ 3 larger compared to a "canonical" divertor. Following Ref. [4], we call this configuration a "SF-plus" configuration.

Specific examples in Ref. [4] were given for simple magnetic geometries The aim of this paper is to demonstrate that the SF concept will also work for a strongly shaped plasma. The other set of issues considered in the present paper relates to the possible presence of the toroidal current near the null-point.

To find a set of divertor coils for a system with a strongly shaped plasma, we use the following strategy. We start from a configuration with the flux surfaces similar to the desired ones in the plasma core. Then we identify the point where the poloidal field null is desired and introduce divertor coils generating this null, be it first or second order. This, of course,

somewhat changes the shape of the flux surfaces in the plasma core but, if the divertor is compact, the change is modest (except for the surfaces close to the separatrix).

We will illustrate this procedure in the limit of a low-aspect-ratio tokamak, replacing it by a "rectified" torus. In this "rectified" geometry, the direction of the plasma current and the current in poloidal field coils is z, with the axes (x, y, z) forming the right-hand triplet. The generalization to the toroidal geometry is straightforward but leads to lengthy equations. General properties of a toroidal field have been discussed in a review paper [5].

The magnetic field in our geometry can be characterized by the flux function $\Phi(x,y)$ (the z component of the vector potential), so that $B_x = -\partial\Phi/\partial y$, $B_y = \partial\Phi/\partial x$. Denote by Φ_0 the flux function generated by all the currents except those flowing in the divertor coils. We will call this field the "initial field." If the plasma current density in the vicinity of the null-point is small and can be neglected, the flux function in the vicinity of the null point satisfies the Laplace equation,

$$\partial^2\Phi_0/\partial x^2 + \partial^2\Phi_0/\partial y^2 = 0. \tag{1}$$

Assume that we have chosen a point x_0, y_0 as a point where the field null has to be formed. Denote the field components of the initial field at this point as B_{0x} and B_{0y}: $B_{0x} = -\Phi'_{0y}\big|_{x,y=x_0,y_0}$, $B_{0y} = \Phi'_{0x}\big|_{x,y=x_0,y_0}$. To make the first-order null, one chooses the location of the divertor coils and the current in them so as to cancel the initial field at the point x_0, y_0. We will call the field generated by divertor coils the "divertor field."

In order to make a second order null, one has to satisfy a second condition condition: make the curvature of the field lines of the divertor field κ_d to be equal to the curvature of the field lines of the initial field. Accounting for Eq. (1), one can show that the curvature of the initial field is equal to

$$\kappa_0 = \frac{\Phi''_{0xx}\left(B_{0x}^2 - B_{0y}^2\right) - 2\Phi''_{0xy}B_{0x}B_{0y}}{\left(B_{0x}^2 + B_{0y}^2\right)^{3/2}}. \tag{2}$$

As shown in Ref. [4], satisfying both conditions ($B_0 = B_d$, $\kappa_0 = \kappa_d$) requires two coils, which are characterized by 6 parameters: a current and two coordinates each. As it turns out [4], the conditions of a second-order null impose 4 constraints on these parameters, therefore leaving two free parameters and providing significant flexibility.

Here we will use the same arrangement as in Ref. [4]: we will place two divertor coils symmetrically with respect to the normal to the initial flux durface (shown in thin line in Fig. 2), with the current in each coil being $I_d/2$. One can easily show that the magnetic field

strength generated at the point $(x_0 y_0)$ by these coils is equal to $B_d = (2I_d b/c)[b^2 + (d/2)^2]^{-1}$ (in CGS-Gaussian system of units). The curvature of the field line of the divertor field at this point is equal to $\kappa_d = [(d/2)^2 - b^2]/b[(d/2)^2 + b^2]$, $b < d/2$. The sign convention here is such that the curvature has the same sign as κ_0 if the divertor coils are situated below the point x_0, y_0, as they should be, in order not to interfere with the core plasma. The matching conditions are:

$$\frac{2I_d}{c}\frac{b}{b^2 + (d/2)^2} = B_0; \quad \frac{1}{b}\cdot\frac{(d/2)^2 - b^2}{(d/2)^2 + b^2} = \kappa_0. \tag{3}$$

In other words, for a given initial field, one parameter of the divertor coils remains free, e.g., b. (The fact that there is only one free parameter, not two, is related to our choice of a symmetrically-situated coils with equal currents). Still, there is significant freedom in selecting the location of the divertor coils, e.g., in choosing the parameter b. An example shown in Fig.3 corresponds to the SF-plus divertor, with the current $I_d=I_{d0}(1+\varepsilon)$, and $\varepsilon = 0.05$.
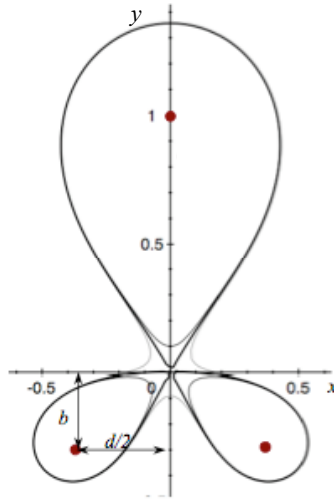

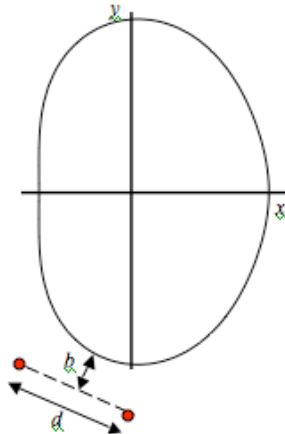Fig. 1. SF divertor in symmetric configuration.

Fig. 2. Generating SF-plus field for the highly shaped plasma. The distance between the divertor conductors (shown in red) is d.
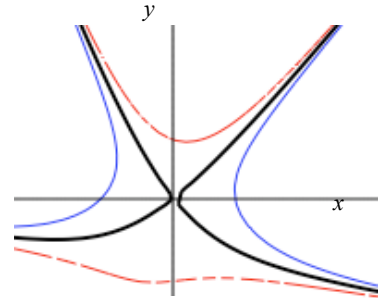

Fig. 3 The shape of the separatrix and two adjacent flux surfaces in the vicinity of the null-point for the asymmetric case.

The case shown in Fig. 3 roughly corresponds to the initial configuration shown in Fig. 2. The origin has been shifted to the null point.

Consider now the role of the plasma current in the divertor area. At a non-zero current density here, the curl of the magnetic field here becomes non-zero. Our approximation of the vacuum field would be correct provided that $|\nabla \times \boldsymbol{B}_p| \ll |\nabla B_p|$, or equivalently, $|j_{zd}| \ll (c/4\pi)|\nabla B_p|$. The subscript "$d$" designates the divertor zone. For the standard X-point divertor, $|\nabla B_p| \sim B_p/b$, where b is the distance between the divertor field conductors and the x point, and B_p is roughly the magnetic field of the plasma current at the X point. It

can be related to the average current density $\bar{j}_z$ in the plasma as $B_p \sim 2\pi a\bar{j}_z/c$, where a is a minor radius of a plasma. Therefore, the condition of a small effect of the plasma current can be formulated as $\mu \equiv j_{zd}/\bar{j}_z << a/2b$. For the examples of the current distribution presented in Refs. [6-8], this inequality holds by a margin of 20 to 30, meaning that, indeed, the current in the divertor zone does not have any significant effect.

In the case of a SF-plus configuration, one should be more cautious, as the magnetic field gradients in the null point are quite small. Here we explicitly add the contribution of the current density in the null-point zone to the flux function Φ_0. We assume the uniform current density and therefore add the term $\Delta\Phi = (\pi j_{zd}/c)\left[(x-x_0)^2 + (y-y_0)^2\right]$. The plots of the separatrix of a SF-plus divertor for several values of the divertor current density are shown in Fig. 4. Thick black line corresponds to $\mu=0$, thin black line to $\mu=0.2$, red line to $\mu=\mu_{crit}=0.273$, and blue line to $\mu=0.3$. One sees that the effect on the separatrix is small even at the parameter μ as high as 0.2. For this particular example, the qualitative change of the configuration occurs only at unrealistically high divertor current density, $\mu>0.273$.
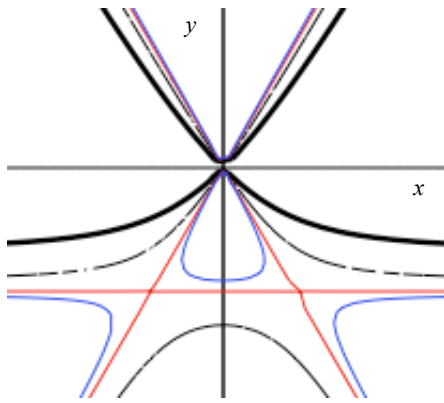

Fig. 4 The structure of the SF-plus field ($\varepsilon=0.05$, $b\kappa_0=0.2$) in the vicinity of the null-point (symmetric case) in the presence of the divertor current.

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